A Technique for the Efficient Computation of the Periodic Green's Function in Layered Dielectric Media

R.M. Shubair and Y.L. Chow

Department of Electrical and Computer Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

ABSTRACT

This paper presents a novel technique for the efficient computation of the *periodic* Green's function in layered dielectric media. This technique is based upon approximating the spectral Green's function by a set of inverse-transformable complex exponential functions. This enables the use of Poisson's summation formula to express the periodic Green's function as a combination of a spectral series and a spatial series each of which is rapidly convergent. The proposed technique is useful as it can be applied to a wide class of problems where periodic structures are to be modeled.

1 INTRODUCTION

When modeling periodic structures in layered dielectric media, one usually encounters a Green's function which converges very slowly. In using the moment method to determine the radiation or scattering from a periodic array, repeated evaluations of the Green's function series are required to fill in the impedance matrix of the structure being modeled. The slow convergence of the series would, therefore, result in a considerable amount of computation time. Previous researchers could only manage to propose accelerating techniques which are limited to the free-space periodic Green's function [1-4]. Thus, a technique which overcomes the slow convergence of the Green's function series and yet can apply to a wide class of problems would be desirable. This paper provides such a technique for periodic structures embedded in general layered media.

The technique uses Prony's method [5,6] to approximate the spectral Green's function by a set of complex exponential functions which are analytically inverse Fourier-transformable. It is the asymptotic behavior of these spectral exponential functions which causes the slow convergence of the Green's function series. To overcome this problem, a procedure similar to that used for the free-space periodic Green's function [4] is implemented. In this procedure, the asymptotic behavior is first subtracted out in the spectral domain and then added back in closed-form in the spatial domain. This is achieved through the use of Poisson's summation formula. The final solution of the periodic Green's function would, therefore, consist of a combination of a spectral series and a spatial series, each of which is by itself rapidly convergent. This makes the overall mixed spectral-spatial summation of the periodic Green's function also rapidly convergent. Hence, the contribution of this paper is to provide a technique that combines the *complex image theory* presented in [5,6] with the accelerating procedure of [4] to efficiently compute the periodic Green's function in layered media.

As an application of the technique presented in this paper, numerical experiments have been conducted to compute the periodic Green's function of a two-dimensional array of dipole point sources printed on a microstrip substrate. The results show the number of terms required for the mixed spectral-spatial summation to converge for different values of the substrate thickness and relative permittivity.

CH3141-9/92/0000-0393$01.00 © 1992 IEEE

1992 IEEE MTT-S Digest

2 THEORY

The periodic Green's function of a two-dimensional array of *phase shifted* point-sources can be expressed in terms of a spectral sum that has the general form

$$G_p = \frac{1}{ab} \sum_m \sum_n \tilde{G}(k_{xm}, k_{yn}).e^{-jk_{xm}(x-x')}.e^{-jk_{yn}(y-y')} \tag{1}$$

$$\text{where} \quad k_{xm} = k_x^i + \frac{2\pi m}{a}, \quad k_{yn} = k_y^i + \frac{2\pi n}{b} \tag{2}$$

also a and b represent the x- and y- periodicities of the structure. k_x^i and k_y^i are wavenumbers associated with the phase shifted plane wave. $\tilde{G}$ represents the *spectral* Green's function of a dipole point source above a general layered media structure as shown in Fig.1. It can be written in the form

$$\tilde{G} = \frac{1}{j2k_z}\left[e^{-jk_z(z-z')} + Re^{-jk_z(z+z')}\right] \tag{3}$$

where R represents the spectral reflection coefficient due to the layered media as shown in Fig.1. Its specific expression can be obtained from the problem under consideration, e.g. [5,6].

It is well known that the spectral series summation in (1) is slowly convergent, especially for the "on-plane" case where $z = z'$. To overcome the slow convergence problem we first approximate the spectral reflection coefficient R by a short series of exponential functions, i.e.

$$R = \sum_{i=1}^{N} A_i e^{jB_i k_z} \qquad (N \le 5) \tag{4}$$

where A_i and B_i are complex coefficients obtained by the application of Prony's method [5,6]. From (4) into (3), $\tilde{G}_{mn}$ becomes

$$\tilde{G}_{mn} = \frac{1}{j2k_z}\left[e^{-jk_z(z-z')} + \sum_{i=1}^{N} A_i e^{-jk_z(z+z'-B_i)}\right] \tag{5}$$

The advantage of the short series approximation of R as given in (4) becomes now evident as the inverse Fourier-transformation of $\tilde{G}_{mn}$ in (5) would now yield a closed-form spatial expression for G_{mn}.

The next step is to introduce an attenuation constant u such that

$$k_z = -j\sqrt{k_{xm}^2 + k_{yn}^2 - k^2} = -j\sqrt{(k_{xm}^2 + k_{yn}^2 + u^2) - (k^2 + u^2)} \tag{6}$$

Clearly from (2) as $m,n \to \infty$, we can take $k_z \to -j\sqrt{k_{xm}^2 + k_{yn}^2 + u^2}$. By replacing this asymptotic value of k_z into (5), the asymptotic behavior of $\tilde{G}_{mn}$ is obtained as

$$\tilde{G}_{mn}^a = \frac{1}{2\sqrt{k_{xm}^2 + k_{yn}^2 + u^2}}\left[e^{-\sqrt{k_{xm}^2 + k_{yn}^2 + u^2}(z-z')} + \sum_{i=1}^{N} A_i e^{-\sqrt{k_{xm}^2 + k_{yn}^2 + u^2}(z+z'-B_i)}\right] \tag{7}$$

By adding and subtracting this asymptotic $\tilde{G}_{mn}^a$ from (1) one obtains

$$G_p = \frac{1}{ab} \sum_m \sum_n (\tilde{G}_{mn} - \tilde{G}_{mn}^a).e^{-jk_{xm}(x-x')}.e^{-jk_{yn}(y-y')} + \frac{1}{ab} \sum_m \sum_n \tilde{G}_{mn}^a.e^{-jk_{xm}(x-x')}.e^{-jk_{yn}(y-y')} \tag{8}$$

Now employing Poisson's summation formula [7] to replace the second spectral sum in (8) by an equivalent spatial sum, one obtains the final form of the periodic Green's function as

$$G_p = \frac{1}{ab} \sum_m \sum_n (\tilde{G}_{mn} - \tilde{G}_{mn}^a).e^{-jk_{xm}(x-x')}.e^{-jk_{yn}(y-y')} + \sum_m \sum_n G_{mn}^a.e^{jk_x^i ma}.e^{jk_y^i nb} \tag{9}$$

where

$$G_{mn}^{a} = \frac{e^{-uR_0^{mn}}}{4\pi R_0^{mn}} + \sum_{i=1}^{N} A_i \frac{e^{-uR_i^{mn}}}{4\pi R_i^{mn}}$$

$$R_0^{mn} = \sqrt{(x-x'-ma)^2+(y-y'-nb)^2+(z-z')^2} \qquad (10)$$

$$R_i^{mn} = \sqrt{(x-x'-ma)^2+(y-y'-nb)^2+(z-z'-B_i)^2}$$

The attenuation constant u is found numerically so the second series in (9) converges most rapidly.

The first series in (9) converges rapidly because two functions are being subtracted out that are asymptotically equal as m and n increase. The second series in (9) converges rapidly because the spatial functions involved decay exponentially as m and n increase. Therefore, the slowly convergent summation of (1) has been successfully replaced by a combination of two rapidly convergent series as given in (9).

3 NUMERICAL RESULTS

As a numerical example, we used the direct sum formula of (1) and the accelerated sum formula of (9) to evaluate the Green's function series of a two-dimensional array of x-directed dipole point sources printed on a microstrip substrate. Results have been obtained for different values of the substrate parameters h and ε_r which represent the substrate thickness and relative permittivity, respectively.

The Green's function example being evaluated here is G_A^{xx} which is the $\hat{x}\hat{x}$-component of the dyadic vector potential. Its specific expression for the microstrip substrate problem is given in the Appendix. Figs.2 and 3 show the number of terms in the accelerated sum formula (9) required to achieve a predefined convergence criteria versus the transverse distance. The results in these figures are shown for the on-plane case where the direct sum formula of (1) has the slowest convergence. For the range of the transverse distance $(\rho - \rho')$ investigated, it is found that the direct sum formula requires over one million terms to converge to machine precision compared to only 2,000 terms at most using the accelerated sum formula. This clearly proves the significant reduction achieved in computation time by utilizing (9) for the evaluation of the periodic Green's function.

4 CONCLUSIONS

This paper presented a new technique for the efficient computation of periodic Green's functions. In comparison with other techniques which are applicable only to the free-space periodic Green's function, our technique can be easily used for periodic Green's functions in general layered media. This makes it of a great potential importance as it can be applied to a wide class of problems where the modeling of periodic structures is required.

Appendix

For a dipole point source located above a microstrip substrate, the $\hat{x}\hat{x}$-component of the dyadic vector potential is given in the spectral domain by

$$\tilde{G}_A^{xx} = \frac{1}{j2k_{z0}}\left[e^{-jk_{z0}(z-z')} + R_{TE}e^{-jk_{z0}(z+z')}\right]$$

where R_{TE} is the TE-wave reflection coefficient due to the grounded slab. Its expression is given by

$$R_{TE} = -\frac{r_{10}^{TE} + e^{-j2k_{z1}h}}{1 + r_{10}^{TE}e^{-j2k_{z1}h}}$$

where r_{10}^{TE} is simply the reflection coefficient of the TE-wave at the dielectric-air interface. It is given by

$$r_{10}^{TE} = \frac{k_{z1} - k_{z0}}{k_{z1} + k_{z0}}$$

References

[1] S. Singh and R. Singh, "Efficient computation of the free-space periodic Green's function", *IEEE Trans. Microwave Theory and Tech.*, Vol.MTT-39, pp. 1226-1229, July 1991.

[2] S. Singh and R. Singh, "On the use of Shank's transform to accelerate the summation of slowly converging series", *IEEE Trans. Microwave Theory and Tech.*, Vol.MTT-39, pp. 608-610, March 1991.

[3] R.E. Jorgenson and R. Mittra, "Efficient calculation of the free-space periodic Green's function", *IEEE Trans. Antennas Propagat.*, Vol.AP-38, pp. 633-642, May 1990.

[4] S. Singh, W.F. Richards, J.R. Zienecker and D.R. Wilton, "Accelerating the convergence of the free space Green's function", *IEEE Trans. Antennas Propagat.*, Vol.AP-38, pp.1958-1962, Dec. 1990.

[5] J.J. Yang, Y.L. Chow, G.E. Howard and D.G. Fang, "Complex images of an electric dipole in homogeneous and layered dielectric between two ground planes", to appear in *IEEE Trans. Microwave Theory and Tech.*, March 1992.

[6] Y.L. Chow, J.J. Yang and G.E. Howard, "Complex images for electrostatic field computation in multilayered media", *IEEE Trans. Microwave Theory and Tech.*, Vol.MTT-39, pp. 1120-1125, July 1991.

[7] R.E. Collin, *Field Theory of Guided Waves*. New York, McGraw Hill, 1991, pp. 759.

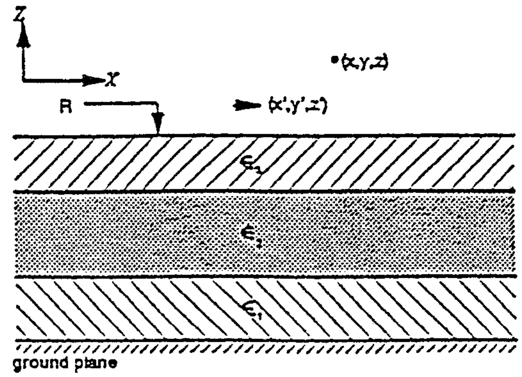

Fig.1 A dipole point source above a layered media structure.

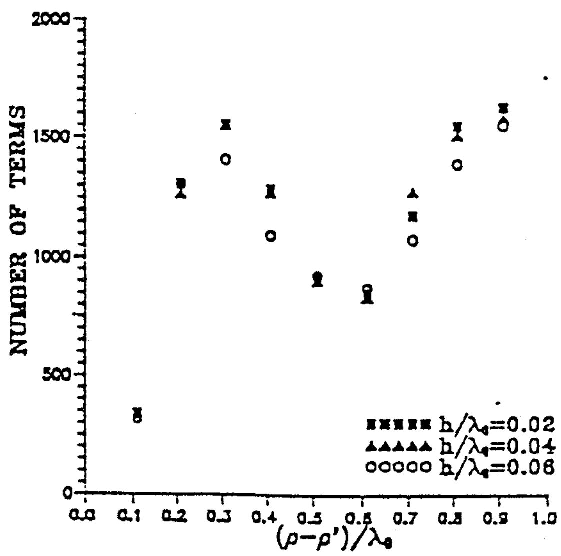

Fig.2 Number of terms versus transverse distance, $f = 30GHz$, $\varepsilon_r = 2.55$, $a = b = 1.1\lambda_0$, $k_x^i = k_y^i = 0$

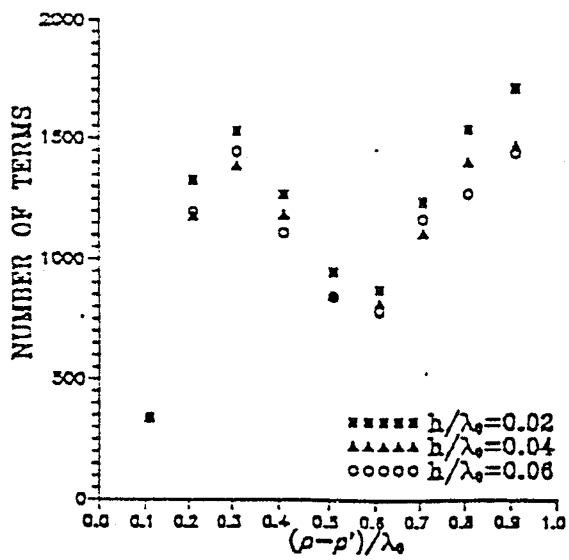

Fig.3 Number of terms versus transverse distance, $f = 30GHz$, $\varepsilon_r = 9.8$, $a = b = 1.1\lambda_0$, $k_x^i = k_y^i = 0$